Macro-Economic Relativity: Government Spending, Private Investment and Unemployment in the USA

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ABSTRACT

A new approach to time series modelling is used to explore how government spending and private capital investment may have influenced the unemployment rate in the USA between 1948 and 1988. The resulting model suggests strongly that the investigation of dynamic relationships between purely relative measures of the major macro-economic variables can help in understanding changes in economic behaviour. It also allows for an initial investigation of the post-1988 period and an analysis of possible reasons for the differences in the investment-unemployment behaviour of the US economy before and after 1988.

KEY WORDS: nonlinear time series analysis; data-based mechanistic modelling; investment and unemployment; relativistic economic variables.

JEL classification: B22; C13; C22

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1. INTRODUCTION

In their usual published form, the macro-economic time series considered in this paper are non-stationary, in the sense that they are characterised by long term stochastic trends which have common characteristics. The major current econometric method for handling such series is the concept of *co-integration* (Engle and Granger, 1987) and the series could be analysed using such cointegration concepts. Here, however, we exploit a rather different, albeit similarly motivated, approach to nonstationary time series analysis based on the definition of economically meaningful and rather natural *relativistic* measures of the variables. In particular, we explore how government spending and private capital investment *measured relative to Gross National Product* (GNP) may have influenced the unemployment rate in the USA (itself a relative measure) between 1948 and 1988. More briefly, we show that our model estimated on the basis of the time series data over this period of time is able to explain the unemployment behaviour over the later, and economically very significant period from 1988-1998.

Previous research (Young, 1994; Young and Pedregal, 1997) has been concerned with the problem of modelling the possible relationship between quarterly measures of seasonally adjusted unemployment rate, GNP and total capital investment (defined here as gross private domestic investment plus Government purchases of goods and services) in the USA over the period 1948 to 1988. The initial research (Young, 1989, 1994) investigated an Unobserved Components (UC) relationship between unemployment rate and GNP, which showed that the perturbations in the logarithm of unemployment rate about its long term trend could be explained well by a first order linear Transfer Function (TF) relationship, using the perturbations of the logarithm of GNP about its long term trend as the assumed exogenous input.

Using a new approach to modelling nonlinear stochastic dynamic systems, we have recently built upon this earlier work and shown that the long term trend in unemployment rate is inversely related to the long term trend in the ratio of total investment (as defined above) to GNP (see Young and Pedregal; 1997: hereafter YP). This then allows for the identification and estimation of a UC relationship for unemployment rate with this relativistic measure of total investment and the perturbations of GNP about its long term trend as the explanatory (exogenous) variables, each entering the UC model thorough TF (distributed lag) models.

In the present paper, we explore further the potential importance of relativity in macro-economics and show how there are advantages in decomposing the total investment into its main constituent parts and then considering separately the measures of both public investment (government spending) and private investment relative to GNP. In this manner, it is possible to remove the perturbational GNP term in the UC model and so eliminate the need to estimate the long term trend in GNP, which represented the main limitation of the earlier UC model.

The UC model obtained in this manner, which now relates purely relativistic variables, is uni-directional but the possibility of feedback relationships existing between unemployment and the relative investment measures cannot be ignored. This possibility is investigated using causality tests applied to a Vector AutoRegressive (VAR) model of the three relative variables: the unemployment rate, y_t (i.e. unemployment as a percentage of the total labour force); public investment relative to GNP, RGI_t; and private investment relative to GNP, RPI_t. This multivariable analysis suggests that the VAR identified feedback effects are insignificant and, even when they are allowed to enter the model, the main conclusions obtained from the uni-directional UC modelling results still apply.

Finally, the paper looks briefly at recently acquired data over the period 1988-1998 and shows how the model, based only on the 1948-1988 data and without re-estimation of the parameters, is able to explain rather well the recent reduction in unemployment over the period 1991-1998, following from the significant changes in RPI_t and RGI_t over this same period.

2. DATA-BASED MECHANISTIC MODELLING

Over the last few years, the first author has developed a new approach to linear/nonlinear systems analysis which he has termed *Data-based Mechanistic* (DBM) modelling (see e.g. Young and Runkle, 1989; Young and Minchin, 1991; Young and Lees, 1993; Young, 1993; Young and Beven, 1994). This is a time series approach which attempts to extend conventional, data-based time-series methodology in a manner which enhances the model builder's ability to interpret the identified model in physical, biological, ecological or, in the present context, socio-economic terms. In the DBM approach, the model *structure* is first obtained by a process of statistical inference applied to the time-series data and based on a given general class of dynamic UC models whose (possibly time variable or state dependent) parameters are estimated using a special form of recursive least squares parameter estimation (closely related to Kalman Filtering) and Fixed Interval Smoothing (FIS: see e.g. Young, 1984, 1988, 1989; Ng and Young, 1990, and the references therein).

Since a full description of the DBM methodology and its application in other scientific areas appears in the earlier papers cited above, it will suffice to concentrate here on its application to the problem at hand, namely of modelling the changes in unemployment rate in the USA over the post second World War period on the basis of the quarterly macro-economic series for the USA shown in Fig. 1, and to assume that the interested reader will consult the previous references for theoretical and algorithmic details.

3. EXPLORING THE RELATIONSHIP BETWEEN UNEMPLOYMENT, INVESTMENT AND GNP IN THE USA 1948(2)-1988(2)

The top graph in Fig. 1 is a plot of the quarterly variations in the unemployment rate, y_t, for the USA over the period 1948(2) to 1988(2), a total sample size of $N = 161$. Below this are graphs of the quarterly variations in GNP, G_t; Government spending on goods and services, GI_t, private capital investment, PI_t, and consumption, C_t, over the same period of time[1]. All of these variables are nonstationary in the mean and, at first sight, there is little apparent relationship between them, except that all have experienced a predominant, upward, long term trend since the end of the second world war. Even in the case of unemployment rate (which, it will be noted, is inherently a relativistic variable) there has been a clearly discernible rise in mean level, particularly over the second half of the data since 1970. However, the series is dominated by the shorter term changes, which are much larger and more erratic. The objective of the present analysis is to obtain a model for the variations in the unemployment rate in terms of the other variables, starting from the results of previous analyses of these data (Young, 1994; YP, 1997).

(INSERT FIGURE 1)

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[1] All data were obtained from Citibase. Using the Citibase mnemonics for the series, the precise definitions of the variables are LHUR (Unemployment rate), GNP82 (GNP), GC82 (Consumption), GGE82 (Government Purchases of Goods and Services), GPI82 (Private Investment). Total investment were obtained by the addition of GGE82 and GPI82.

3.1 Previous Models

Young (1994) investigated a UC relationship between $\log_e(y_t)$ and $\log_e(G_t)$ which showed that the perturbations in $\log_e(y_t)$ about its long term trend could be explained very well by a first order linear TF relationship, using the perturbations of $\log_e(G_t)$ about its long term trend as the assumed exogenous input. More recently, using normal rather than logarithmically transformed data, YP(1997) have exploited the DBM approach, and in particular FIS estimation, to show that the long term trend in unemployment rate can be associated with the changes in the relative investment ratio $RI_t = I_t/G_t$, shown in Fig. 2, where I_t is the total capital investment (as defined above).

(INSERT FIGURE 2)

Although the long trend in unemployment is explained very effectively in the YP model by RI_t, the short term (and quite large) fluctuations of unemployment rate about this long term behaviour are still being related mainly to the perturbations ∇G_t of GNP about its long term trend T_t^g. This is rather unsatisfactory on two counts: first, unlike the other two variables, ∇G_t is not a relativistic variable; second, the use of ∇G_t necessitates the introduction of a stochastic Integrated Random Walk (IRW) model for the trend. Both of these factors limit the clear economic interpretation of the model and, in this sense, the model does not quite conform with the basic philosophy of DBM analysis, which requires that the model should have an acceptable physical (here macro-economic) interpretation. It would be far better, therefore, if this second explanatory variable could be replaced by another relativistic macro-economic indicator, so obtaining a model that relates purely relative measures of economic behaviour.

3.2 An Improved Model

Fortunately, a simple and elegant solution is revealed if we consider further the relativistic definition of RI_t and consider other, similarly defined, relativistic measures of macro-economic variables, as shown in figs. 3 and 4. The bottom two graphs in Fig. 3 are, respectively, plots of RC_t, the ratio of total consumption (as defined above) to GNP; and $C2I_t$ the ratio of consumption to total investment. These can be compared with the graph of RI_t at the top, which has been inverted for comparative purposes. It is clear from these plots that the conversion of the macro-economic variables into relativistic measures, as suggested in a quite objective manner by the DBM nonlinear analysis, has exposed aspects of the US economy that are not readily observable from the basic plots of the variables in Fig. 1. In particular, the underlying changes in the US economy from around 1970 become clear, with the reduction in total investment spending being coincident with rises in relative consumption and the consumption/investment ratio. The latter is particularly interesting and is discussed later in section 6.[2]

(INSERT FIGURES 3 & 4)

Fig. 4 provides the main justification for modifying the earlier YP model. It shows the variables obtained when the components of the total investment variable I_t are considered separately in a relativistic manner to yield relative government spending $RGI_t = GI_t/G_t$ and relative private capital investment, $RPI_t = PI_t/G_t$. It is clear from these plots that the reduction of RI_t that contributes so much to the explanation of the long term rise in unemployment in the YP model appears to be due to the decline of public rather than private investment relative to GNP: in particular, standard statistical tests (Augmented Dickey-Fuller and Chow structural change) show unambiguously that the mean level of RGI_t (shown dotted) declined significantly from a roughly constant level of 24.48±0.69% of GNP (i.e. 0.2448±0.0069) in the period 1955-1969, to 20.07±0.72% of GNP, in the period 1973-1988; meanwhile relative private investment RPI_t, whilst very volatile in the short term, remained at a roughly constant mean level (16.68±1.45.% of GNP) over the whole period.

It is this volatility of RPI_t that appears important in relation to the short term variations of the unemployment series about its long term trend in the period 1970-1988: in particular,

[2] As an anonymous referee suggested, the relative measure of current account to GNP could also be considered since the total spending (or aggregate demand) includes the current account, as well as consumption, investment and government spending. However, the relationship between current account and unemployment is relatively weak when compared with that between the investment variables and unemployment and so it has not been included in the current analysis.

standardised plots of RPI_t and ∇G_t reveal a close resemblance in the characteristics of both variables and suggest that RPI_t can replace ∇G_t in the YP model. This not only has the desirable effect of eliminating the IRW trend equation, as required, but it also leads to a meaningful macro-economic interpretation of the model. The resulting model then takes the form,

$$y_t = C + \frac{b_1}{1+a_1 L} RGI_t + \frac{b_2}{1+a_1 L} RPI_t + \frac{1}{1+c_1 L + c_2 L^2} e_t \qquad e_t \sim NID(0, 0.101) \tag{1}$$

where L is the backward shift operator, i.e. $L^n y_t = y_{t-n}$. The parameter estimates, standard errors and diagnostic test results are listed below in Table 1[3]. Here, R^2 denotes the standard Coefficient of Determination (COD) defined in terms of the one step ahead prediction errors; while R_T^2 is an alternative COD measure defined in terms of the TF model response errors $\tilde{y}_t = y_t - \hat{y}_t$, where $\hat{y}_t$ is defined as follows,

$$\hat{y}_t = \hat{C} + \frac{\hat{b}_1}{1+\hat{a}_1 L} RGI_t + \frac{\hat{b}_2}{1+\hat{a}_1 L} RPI_t \tag{2a}$$

or, in equation terms,

$$\hat{y}_t = -\hat{a}_1 \hat{y}_{t-1} + \hat{b}_1 RGI_t + \hat{b}_2 RPI_t + \hat{c} \tag{2b}$$

where the constant $\hat{c} = (1+a_1)\hat{C}$ and the 'hats' denote the estimated values of the parameters. Here, $\hat{y}_t$ can be interpreted as an estimate of the 'noise free' output from the model, that is the output due to the exogenous inputs alone, with no reference at all to the measured unemployment y_t. This variable is considered to be of primary importance in most systems and control modelling studies because it shows how well the TF model is relating the exogenous inputs (which are often manipulatable control inputs in the systems context) to the output. Thus

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[3]Note that if consistent units are used throughout this model (i.e. all variables measured in either fractions or percentages), then the only difference is that the b_1 and b_2 estimates, their standard errors, and the steady state gains are simply multiplied by 0.01: i.e., $\hat{b}_1$=-0.1508±0.017; $\hat{b}_2$=-0.3298±0.028; G_1=-0.6764 and G_2=-1.4792.

R_T^2 is usually a more discerning measure of model fit than R^2 and is often the favoured measure in systems modelling.

(INSERT TABLE 1)

The results shown in Table 1 were obtained using RIV identification and estimation (an optimal form of Instrumental Variable estimation developed for systems modelling; see e.g. Young, 1984). It should be noted, however, that two other, better known, estimation methods (Exact Maximum Likelihood (ML); and the Prediction Error Minimisation (PEM) algorithm in the Matlab™ identification toolbox) yield very similar models, thus demonstrating that the model is quite independent of the estimation method used in the analysis. The R_T^2 values vary from 0.8838 for RIV and PEM to 0.8808 for ML; whilst R^2 ranges from 0.9655 for PEM through 0.9644 for RIV to 0.9639 for ML. Also shown in the results below the Table are: the Jarque-Bera, Q(4) and Q(8) diagnostic statistics (See Jarque and Bera, 1980; Ljung and Box, 1978), which verify the statistical adequacy of the models in each case; the TF steady state gains, G_1 and G_2 (long term multipliers); and finally, the common time constant T_c associated with the $1+\hat{a}_1 L$ denominator polynomial.

Figs. 5 and 6 confirm graphically the satisfactory nature of the model residuals (one step ahead prediction errors) e_t: Fig. 5 is a plot of the residuals, together with their associated simple and partial autocorrelation functions (acf and pacf, respectively); and Fig. 6 exhibits plots of the cross correlation functions (ccf's) between e_t and the two inputs, RGI_t and RPI_t. It is clear that the e_t are not only serially uncorrelated but they are also not significantly correlated with the exogenous inputs, as required. Finally, nonlinearity tests (Billings and Voon, 1986; White, 1980; Engle, 1982) indicate that there is no significant residual nonlinearity, which has been satisfactorily purged from the data by the introduction of the two nonlinear investment/GNP ratios.

(INSERT FIGURES 5 & 6)

The top graph in Fig. 7 is a plot of the unemployment series y_t compared with the RIV estimated model output $\hat{y}_t$ from equation (2). The graph below this compares the one step ahead predictions from the RIV model with the unemployment series: not surprisingly, the fit to the data in this case is very close, given the high $R^2 = 0.965$. Note that the modelling errors are larger before 1960 than after this date: this is almost certainly caused by additional factors that have not been taken into account in the present analysis: for instance, price controls, the Korean War and the Treasury-Fed. accord. This suggests that further improvement in the model may be possible if such additional factors are considered over this earlier period. The possible introduction of other explanatory variables is mentioned later in section 6 of the paper.

(INSERT FIGURE 7)

Fig. 8 shows the model estimated contributions that the exogenous inputs are making to explaining the variations in unemployment: at the top, for reference, is a repeat plot of $\hat{y}_t$ compared with the unemployment series; whilst, below this, are the estimated 'noise free' outputs of the two TF's and the constant term c, that additively produce $\hat{y}_t$. Over the period 1970-1988, the model suggests that the main effect of the relative government spending input RGI_t plus the constant c, in the middle plot, is to model the long term upward variations; while the relative private investment input RPI_t explains the shorter term, and rather large, fluctuations in unemployment rate. An important caveat is necessary here, however: these specific conclusions apply over this period of time (1970-1998), since the variations in the model predicted output are consequent only upon the changes in the investment variables RGI_t and RPI_t over this same period. At other times, one or the other of the investment variables could have a dominant effect on the modelled unemployment behaviour, so masking the effect of the other. Indeed ,we will see later in section 5 that exactly such a situation develops in the years after 1988.

(INSERT FIGURE 8)

In this latter connection, it is interesting to explore what the model tells us about the relative effectiveness of public and private investment in affecting the level of unemployment. Referring to the two TF's in the model (2), we see that the steady state gain (equivalent to the 'long term multiplier' in the semantics of economics) of the TF between RPI_t and the unemployment rate ($G_2 = \hat{b}_2 / (1 + \hat{a}_1) = -147.9$) is 2.19 times the steady state gain between RGI_t and the unemployment rate ($G_1 = \hat{b}_1 / (1 + \hat{a}_1) = -67.6$). Since the steady state gain is the steady level that the output of the TF concerned achieves following a sustained unit step in the input variable, this means that a 0.01 (1%) permanent increase in the relative level of private investment would lead to a permanent reduction of 1.48% in the unemployment rate; while a similar permanent increase in the relative level of Government spending would only lead to a reduction of 0.68%.

Of course, the problem from an economic management standpoint is that, while Government is able to exercise some substantial control over its relative level of its spending, it has much less control over private capital investment, which is largely dependent upon the performance of the private economy. In this connection, it would appear that, between 1970 and 1988, there was no permanent increase in the underlying relative level of private capital investment to match the apparently permanent decrease in the relative level of Government spending that occurred over this period. On the other hand, as we shall see in section 5, RPI_t has increased radically after 1991 and this has lead to a considerable fall in unemployment, despite further reductions in RGI_t.

3.3 How well does the model forecast the Unemployment Rate?

Our primary objective in this paper is to obtain a data-based model of unemployment rate that may help in better understanding the economic mechanisms that affect this most important social indicator. However, as described above, the model has been identified and estimated in a time series UC model form and so it is possible to use it also for forecasting the unemployment rate. Naturally, this can prove useful in model evaluation terms since, if reasonable multi-step ahead forecasts are obtained, it should engender more confidence in the efficacy of the model.

With additional model evaluation in mind, we have set the model a very difficult challenge and used it to forecast the unemployment rate over the last ten year period of the data set, from 1978 to 1988, with the model parameter estimates based *only* on the data up to and including the last quarter of 1977. Two forecasts are generated in this manner: the first a true, out-of-sample (*ex-ante*) forecast, in which the exogenous relative investment variables required by the model are themselves forecast ten years into the future on the basis of their past behaviour; and the second, a reference forecast, when these exogenous variables are assumed known over the ten year period. Effectively, this second forecast illustrates the 'best possible' results that could be obtained from the model and is useful for comparative purposes.

The forecasting results are shown in Fig. 9. Here, in the *ex-ante* situation, the ten year ahead forecast, shown by crosses, is accomplished by converting the model (1) into a discrete-time, stochastic state space form and then incorporating it into a Kalman filter algorithm, with the explanatory variables RGI_t and RPI_t modelled and forecast *separately* over the ten year period using a *Dynamic Harmonic Regression* (DHR) time series model (see e.g. Young et al, 1989; Ng and Young, 1990) with its hyper-parameters optimised in the frequency domain (see Young, 1994 and Young *et al*, 1996). The forecast based on the actual future levels of RGI_t and RPI_t is shown as a full line. The confidence levels on both of these forecasts are naturally quite large and so they are not shown explicitly to avoid confusion on the graph. For comparison, the actual unemployment statistics up to 1988 which, it must be emphasised again, were not used at all in producing the forecasts, are shown by circles.

(INSERT FIGURE 9)

It is clear that the forecasts are quite good and so give some additional confidence in the model, particularly when it is note that the ex-ante forecast in Fig. 9 is dependent upon the forecast of the very volatile RPI_t variable, which itself represents a formidable forecasting problem. Consequently, the forecasting performance depends upon whether, at the forecasting origin, the RPI_t can be forecast well into the future based on its past behaviour. Since private investment is notoriously difficult to forecast, certainly over such a long, ten year ahead period, we can assume that the forecasting performance of our model will not always be as good as that illustrated in Fig. 9. It is interesting, therefore, to see how the model performs with much poorer estimates of RGI_t and RPI_t. For example, the forecast obtained by maintaining RGI_t and RPI_t at constant levels into the future, based on their local mean values in the last quarter of 1977, is shown as the dashed line on Fig. 9. Whilst, quite naturally, this does not forecast the perturbations in unemployment rate arising from the future, un-predicted volatility in RPI_t, it does capture very reasonably the underlying level of unemployment over the ten year forecast period. Clearly other forecasts could be generated using various 'what-if' scenarios for RGI_t and RPI_t.

Finally, although the forecasting performance in Fig. 9 is quite good in both cases, it must be emphasised that the model (1) has not been obtained specifically for such forecasting applications and it is quite possible that it could be modified further to yield even better multi-step ahead forecasts if this was considered to be the main objective of the analysis.

4. MULTIVARIABLE ANALYSIS: IS THERE EVIDENCE OF FEEDBACK?

One criticism of the analysis in the previous section is that it assumes the exogeneity of the two explanatory inputs RGI_t and RPI_t in model (1); in other words, it is assumed that no significant feedback effect exists between unemployment rate and these input variables. From an economic standpoint, however, this assumption could be questioned, since it seems possible that the level of unemployment might affect both the Government's spending policy and the level of private capital investment. It is essential, therefore, that we consider further whether there may be any evidence in the data of *significant* feedback influences that might affect the conclusions derived from the model.

There are a number of ways of considering this problem but the most straightforward is to make no *a priori* assumptions about the direction of causation by modelling the data vector $\boldsymbol{X}_t = [y_t \;\; RGI_t \;\; RPI_t]^T$ as a Vector Auto-Regressive (VAR) process, which then allows for dynamic interaction between all the variables in $\boldsymbol{X}_t$, and so permits feedback effects. Following standard practice (see e.g. Priestley, 1989), the Akaike Information Criterion (AIC) identifies the following third order, VAR(3) process,

$$\boldsymbol{X}_t = \boldsymbol{c} + A_1\boldsymbol{X}_{t-1} + A_2\boldsymbol{X}_{t-2} + A_3\boldsymbol{X}_{t-3} + \varepsilon_t \qquad \varepsilon_t \sim \text{NID}(\boldsymbol{0},\, \Sigma_\varepsilon)$$

where $\boldsymbol{c}$ is a vector of constants, $A_i, i = 1,2,3$, are 3x3 dimensional matrices of parameters and Σ_ε is the covariance matrix associated with the white noise input vector ε_t. Estimation of this model against the data, yields the parameter estimates shown below in Table 2, where

$dA_i, i = 1,2,3$, are the standard error matrices associated with the parameter matrices $A_i, i = 1,2,3$; and the $R_i^2, i = 1,2,3$, are the standard COD's associated with the one step ahead prediction errors of each equation in the VAR model. Extended versions of the Jarque-Bera and pormanteau Q statistic for autocorrelation of the residuals in the multivariate context are also shown below[4], and these again verify the statistical adequacy of the model.

(INSERT TABLE 2)

As can be seen from the R_i^2 values, this VAR(3) model explains the data quite well. Moreover, simulation of the model shows that its dynamic characteristics are consistent with those of the uni-directional TF model (1); i.e. the effects of the explanatory variables on the unemployment rate are very similar to those suggested by the model (1). Most importantly in the present context, however, it is possible to apply *Granger Non-Causality* tests (e.g. Lütkepohl, 1991, chapter 3) to evaluate the most probable directions of causality revealed by the model. The results of this analysis are shown below in Table 3. In this Table, the null hypotheses relate directly to joint significance tests of the sets of parameters shown in Table 2. For example, to test if RGI_t and RPI_t does not cause y_t, the significance of the parameters in the positions (1,2) and (1,3) of all the A_i matrices has to be tested. The table reports two statistics with F and χ^2 distributions (see Lütkepohl, 1991).

(INSERT TABLE 3)

The conclusions from Table 3 are quite clear: there is strong evidence of a relation from RPI_t and RGI_t to the unemployment rate; while feedback between the unemployment rate and these explanatory variables is rejected in both cases by a wide margin (see rows 5 and 6 of Table 3). There only exists some possibility of a weak feedback in line 4 of Table 3, where the hypothesis cannot be rejected at the 1% significance level, but would be rejected at the 5% level. All these results are coherent with those of the individual t-tests on the parameters of the

[4] The tests used here are described in chapter 4 of Lütkepohl (1991)

VAR(3) model. Actually, all the parameters of the three matrices in positions (2,1) and (3,1) are not significant individually at the 5% level of significance.

On the advice of an anonymous referee and to gain additional insight into this aspect of the analysis, Hausman specification error tests were also carried out (see e.g. Holly, 1982; Greene, 1997) in order to check on possible endogeneity of the inputs. The test consists basically of a comparison between the estimates shown in Table 1 and 'two stage' estimates obtained by replacing the inputs in equation (1) by the 'noise-free' inputs taken from the second and third equations of the VAR(3) model. These estimates are consistent, even when the inputs are endogenous, while the estimates in Table 1 would be consistent only if the inputs are exogenous.

The results of the Hausman tests, as shown in Table 4, confirm the results obtained in the causality analysis. Firstly, if both noise-free inputs are used in equation (1), individual tests of exogeneity for RGI_t and RPI_t (rows 1 and 2 of Table 4), or a joint test for both of them (row 3) are possible. Secondly, if equation (1) is estimated including the original data for one of the inputs and the other one based on the VAR(3) model, then this allows for an endogeneity test on the noise-free input included in the equation (rows 4 and 5 for RGI_t and RPI_t, respectively). These results suggest that there is no evidence of any *important* problems of endogeneity in both inputs (although depending on the significance level, RGI_t could be considered either endogenous or exogenous).

(INSERT TABLE 4)

To conclude, the Hausman tests indicate no real evidence of any endogeneity problems in relation to the inputs of equation (1); and Granger causality tests indicate no strong indication of feedback effects . As a result, it is reasonable to consider that the data are described adequately by the simpler uni-directional TF model (1) and consider the economic implications of this model, which are much more transparent than in the alternative, multivariable VAR(3) model. Note again, however, that this conclusion is confirmed also by the dynamical properties of the VAR model in Table 3, which are very similar to equation (1), implying that feedback effects, even if they are present, are very small and do not effect the conclusions of the present paper.

5. POST 1988 BEHAVIOUR

Very recently, during the reviewing and revision of this paper, we have been able to obtain additional quarterly data over the period 1988-1998. Unfortunately, the definition of the Citibase archived variables has been changed in relation to the 1948-1988 data considered in previous sections of this paper and so it has not been possible to employ these new series directly to generate an entirely consistent set of data for the whole of the 1948-1998 period. However, since the major differences between the two sets of data in recent years lie in the underlying levels (mean values) of the series, we have been able to adjust the background levels of the new data (without any other modification) so that the old (1948-1988) and the new (1988-1998) series are congruent in level[5], and then consider how well the model (1) is able to explain the unemployment behaviour over the entire period including, most importantly, the period 1988-1998.

Figure 10 shows resulting plots of the unemployment rate (upper plot: full line) and relative investment series RGI_t and RPI_t (lower two plots) over this extended period. Also shown in the top plot is the output $\hat{y}_t$ (dashed) obtained from equation (2), where it must be emphasised that all the model parameter estimates are those reported in Table 1, which were estimated from the time series data over the period 1948-1988, without any reference to the post-1988 series. Also, note again that $\hat{y}_t$ is generated from the exogenous inputs (the relative investment variables RGI_t, RPI_t, and the constant term, c) alone and is not dependent at all on the measured unemployment y_t.

Referring to Fig. 10 over the post 1988 period, we see that, after 1991, RGI_t fell steadily from its previous rather stable level of around 0.2 to 0.157; while, in contrast, the RPI_t increased, quite remarkably, from 0.16 to 0.219, a level marginally greater than its previous

[5] The levels are modified so that, in each case, the first value of the post-1988 series is equal to the average of the last 3 quarters of pre-1988 series. This yields constant level adjustments to the post-1988 series of +0.033, -0.0055 and +0.0383 for y_t, RGI_t and RPI_t, respectively. Note that these adjustments are very small and no other adjustments are made to the series.

maximum of 0.215 achieved 48 years ago in 1950! In other words, while relative Government spending as a percentage of GNP fell to an all time low of 15.7% in 1998 (lower even than the previous lowest value of 17.8% in 1948), private capital investment in 1998 increased to 22% of the GNP, compared with the previous quasi-cyclical maxima over more recent times of 19% in 1973, 18.5% in 1979, 18.9% in 1984 and 19.3% in 1988.

In reaction to these unprecedented movements in RGI_t and RPI_t, the unemployment rate y_t reached a local maximum of 7.63% in 1992 and, thereafter, fell steadily to reach a level of 4.3% in 1998, the lowest level since 1969. This behaviour is entirely consistent with that predicted by our model in equations (1) and (2) on the basis of these changes in the investment variables, as illustrated by the close relationship between $\hat{y}_t$ and y_t in the upper plot of Fig. 10. Although the RGI_t falls by 4.3% of GNP and so introduces an increasing component to $\hat{y}_t$, this is more than compensated by the still larger increase of 5.9% in RPI_t and the fact that the steady state gain (multiplier effect) associated with the RPI_t is 2.19 times that of the RGI_t steady state gain, as discussed previously in section 3.2. In other words, since our model suggests that RPI_t changes are over twice as effective as RGI_t changes in influencing unemployment rate, it is the positive changes in RPI_t over the period 1991-1998 that entirely mask the much smaller negative effect of the reduction in RGI_t. As a result of these major changes in the investment variables, the US economy appears currently (1998) to be in a very good economic situation, with almost unprecedented high levels of relative private investment and low levels of unemployment.

6. DISCUSSION

Before discussing the modelling results presented in previous sections of the paper any further, it is important to consider first the data used in the model development. In a very real sense, the modelling studies simply help to confirm that the quite clear, visible relationship between the macro-economic variables plotted in Figs. 3, 4 and 10 can be justified statistically and so provide a basis for economic inference. In particular, the variations in the relativistic macro-economic variables, y_t, RGI_t, RPI_t, RC_t and $C2I_t$, as revealed objectively in the nonlinear identification stage of the DBM analysis (YP, 1997), help to expose significant developments in the US. economy that are not so readily discernible in the basic macro-economic variables plotted in Fig. 1. In other words, it seems that the co-movement in these ratio variables is a clearer indicator of economic behaviour and performance (here represented by the changes in unemployment rate y_t, although other relativistic variables could be considered in a like manner) than the more commonly published level variables (GI_t, PI_t and C_t) on which they are based.

The ratios RGI_t and RPI_t that are so important to our model development have been discussed quite fully in section 3.2. However, it is interesting to consider further the other ratios shown in Fig. 3: namely the relative consumption variable RC_t or, more particularly, the ratio of consumption to total investment $C2I_t$. Between 1955 and 1970 this latter ratio was relatively stable, with a mean value of 1.456±0.042; between 1970 and 1974, however, it rose to a new level of 1.561±0.016; and finally, over the period 1974-1988 it was much more volatile with a mean of 1.745±0.07. In other words, there appears to have been an approximate 20% shift in resources from total investment to consumption over this historical period.

If we consider the ratio of consumption to public investment alone, then the picture is even more startling: this ratio changes from 2.43±0.066 between 1955-1970, to 3.248±0.057, a rise of approximately 34%. At the same time, it is clear that, despite political predictions to the contrary, the relative private investment ratio RPI_t *up to 1988* did not expand in a sustained manner to fill the gap created by the relative reduction in Government spending. Rather it became much more volatile: increasing at times of economic boom but then sinking to very low levels (at times below 0.14) relative to the highest values achieved in recent times of around 0.19. But even the increases, when they occur over this time period, never approach the level required to compensate for the massive reductions in RGI_t from a maximum of 0.259 to a minimum of 0.189.

As we have seen in section 5, however, a very significant change in the behaviour of RPI_t occurs after 1988 when the sustained increases in this variable over this later period more than compensates for the reductions in RGI_t. In the context of the present paper, however, the important point is that our model, estimated from the data over the period 1948 to 1988, continues to predict well the effect of the changes in RPI_t and RGI_t on unemployment over the later period 1988-1989. Indeed, it shows that in the situation of very high, maintained levels of RPI_t, unemployment can fall to historically low levels even when RGI_t is decreasing. The economic question, however, is whether this situation can be sustained: in the past, RPI_t has been very volatile and should it suffer a large reduction to its previous low levels of around 0.14 (as a result, for instance, of the current global economic problems and its repercussions), without any concomitant increase in RGI_t, then our model would suggest that the unemployment rate would reach levels of 14%. Whilst this is, of course, a 'worst case' scenario, it does provide food for thought.

Finally, we wish to stress that the main aim of the paper is simply to draw attention to the remarkably constant relationship amongst the subset of relativistic variables considered here over the period 1948-1988. The paper certainly does not seek to represent the whole unemployment phenomena in the USA in any complete, economically meaningful manner, since other relevant variables included in many standard economic studies are not considered explicitly in our analysis: for instance, variables such as the consumer price index, money stock, interest rate, exchange rate and oil prices, as well as effects such as the relative prices of inputs with labour-capital substitution, international trade and jobs being exported. All of these have potential effects on unemployment and clearly should be taken into account in any extension of the model considered here. On the other hand, it is clear from a statistical standpoint that the relative investment variables considered in our model explain and forecast the unemployment time series very well and leave only a little remaining variance to be explained by other potential explanatory variables.

7. CONCLUSIONS

This paper has sought to show that the changes in the unemployment rate of the USA over the period 1948 to 1988, and particularly since 1970, appear to be related in a linear dynamic manner to changes in nonlinear relativistic measures of public and private investment: namely the proportion of the GNP devoted to Government spending on goods and services, RGI_t; and the proportion of GNP accounted for by private capital investment, RPI_t. The resulting model suggests strongly that while increases in RPI_t are more effective in reducing unemployment, it was the very significant and apparently permanent reductions in RGI_t between 1970 and 1988 that seem most related to the underlying, longer term rise in unemployment *over this period*. In other words, despite political and economic hopes to the contrary, the long term reductions in the relative level of Government spending were not compensated by significant long term increases in the relative level of private investment. Rather, the heavy volatility that characterised the movements in RPI_t (and other macro-economic indicators: see Young, 1994) between 1970 and 1988 seems to relate mainly to the large, short term fluctuations in the unemployment rate about its underlying long term level, which increased from around 4% to 7% over this period of time.

In contrast to the period 1948 to 1988, however, the changes in RPI_t over the subsequent period between 1988 and 1998 have been quite different in nature, with relative investment experiencing an almost unprecedented rise to levels marginally greater than those encountered transiently as long ago as 1950, and some 15% greater than the previous local maxima experienced in the economic boom periods of more recent times. Consequently, our model suggests that, unlike the period 1948 to 1988, RPI_t has had a dominating effect on the level of unemployment over the period from 1991, with its sharp rise and associated, much higher, steady state gain (multiplier) leading to reductions in unemployment that almost completely eclipse the very much smaller increases resulting from the reduction in RGI_t. Note, however, that while the model indicates that the special circumstances of this 'super boom' have conspired to reduce the effect of RGI_t on unemployment, Government expenditure has now reduced to levels lower than even those seen previously just after the second World War in the late 1940's. In this situation, our model suggests that the US economy may be vulnerable to any rapid reductions in private investment, unless they are accompanied by concomitant, compensatory increase in the relative level of Government expenditure.

Of course, in explaining and forecasting the macro-economic variables rather well, the model does not *prove* unambiguously that there is any causal link between these relative measures of investment and unemployment and so fully justify the above interpretation, which remains, to some extent, conjectural. On the other hand, it is also clear from the analysis that the possibility of such causality, which has such important economic implications, is very well supported by our analysis and is deserving of further detailed evaluation. In this sense, we hope that this paper will stimulate more detailed modelling studies of macro-economic dynamics in the US. and other World economies that similarly exploit the relativistic approach to macro-economic modelling that we have sought to promote in the present paper. Perhaps also the potential importance of relativistic measures in economic modelling, as revealed in this paper, may lead to an economic theory of relativity that is better able to explain and forecast the dynamic behaviour of the economy than the wide variety of more conventional macro-economic models that abound in the past literature on this most challenging of subjects.

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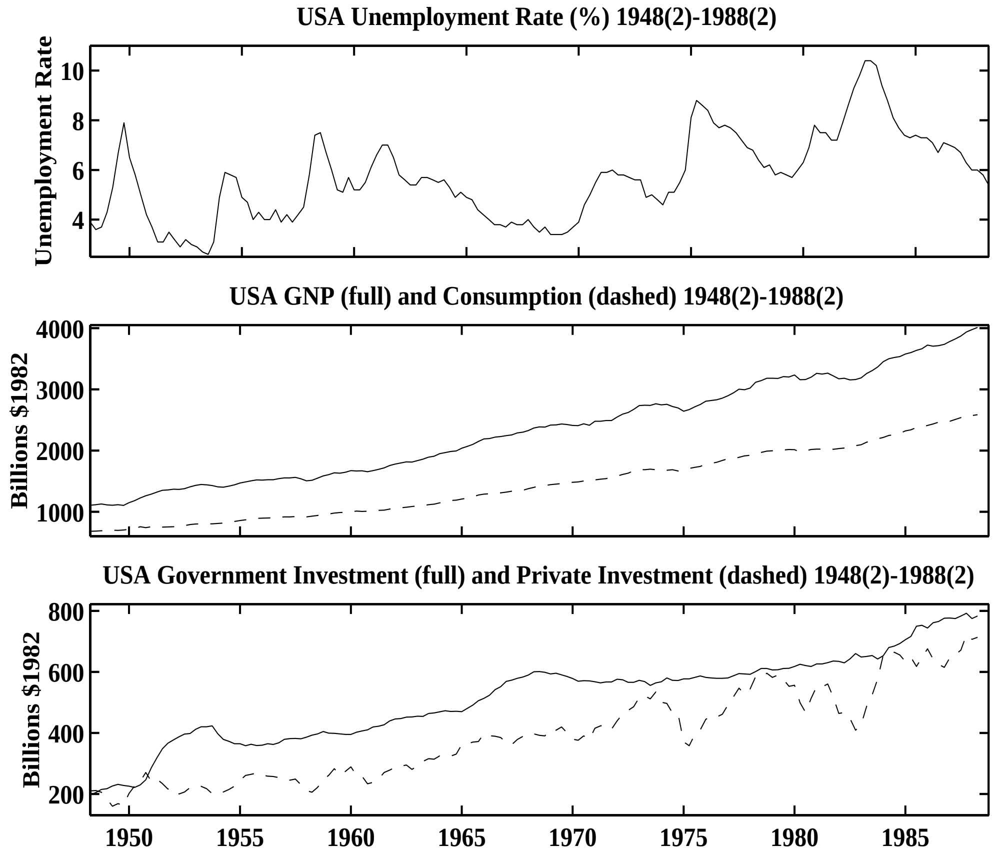

Fig. 1 Selected macro-economic data for the USA 1948(2)-1988(2)

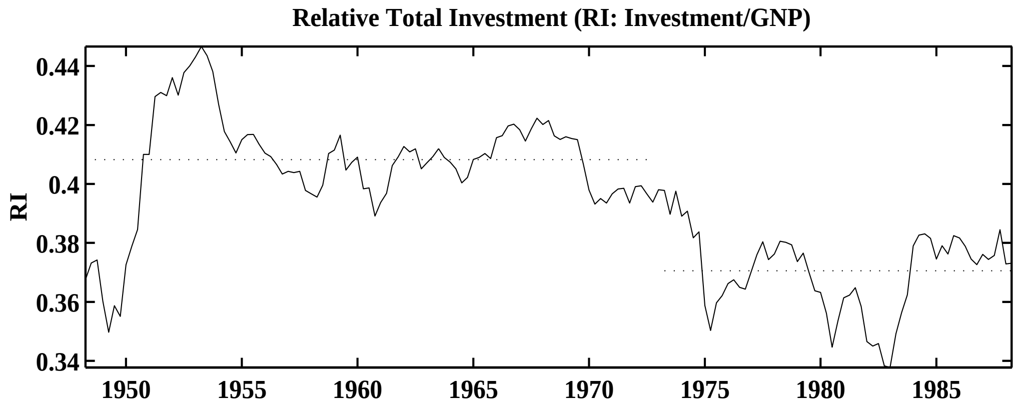

Fig. 2 The Relative Investment variable RI_t

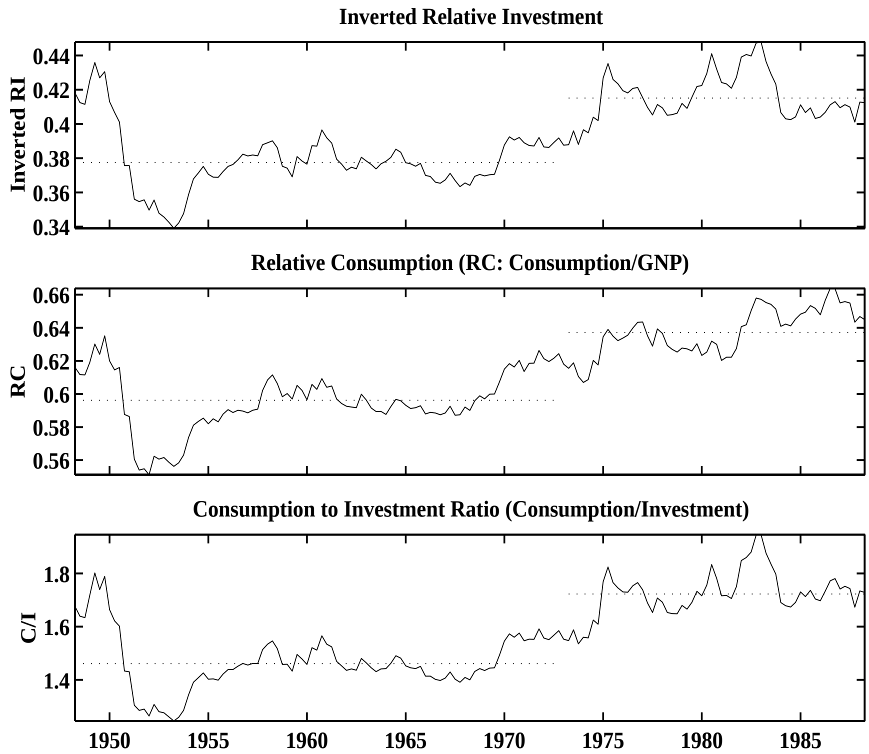

Fig 3 Some relativistic measures of macro-economic variables: Relative Investment, RI_t (inverted), Relative Consumption RC_t; and Consumption to Investment Ratio, $C2I_t$.

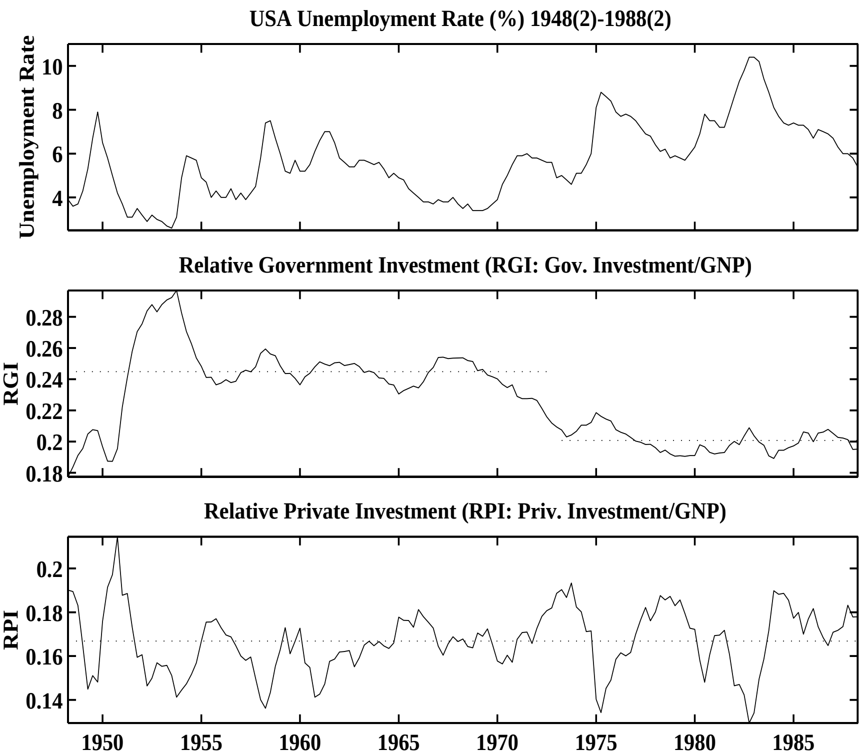

Fig. 4 Unemployment Rate y_t, Relative Government Investment RGI_t, and Relative Private Investment RPI_t.

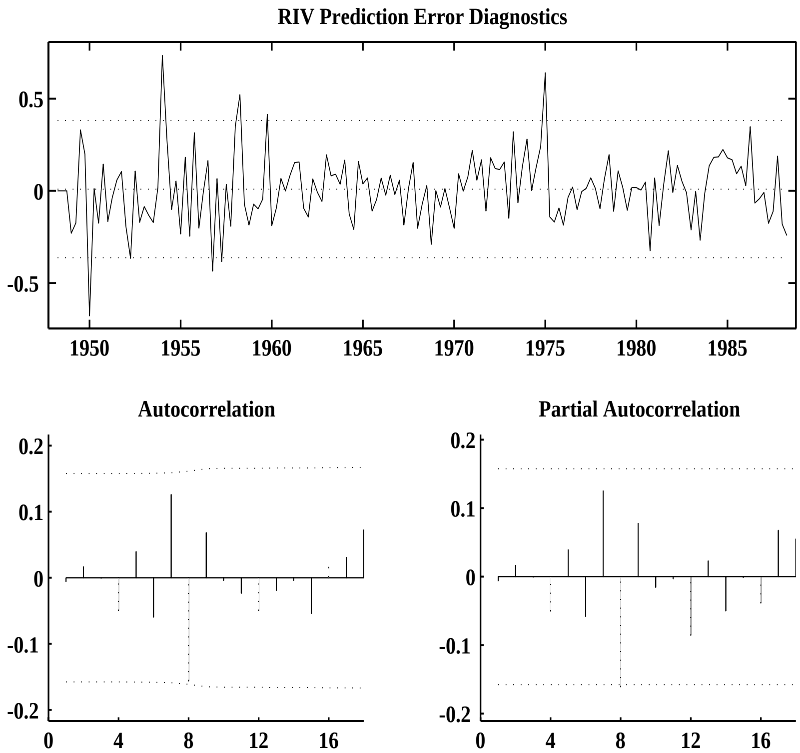

Fig. 5 Final RIV estimation: model residuals (one step ahead prediction errors e_t), together with their simple and partial autocorrelation functions (below)

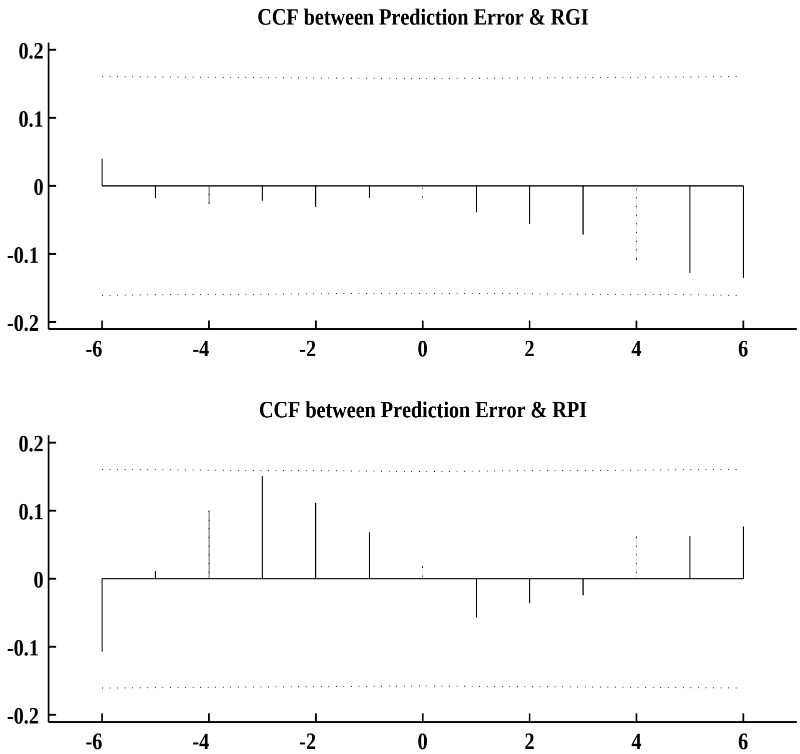

Fig. 6 Final RIV estimation: cross correlation functions between the model residuals e_t and the two input variables RGI_t and RPI_t

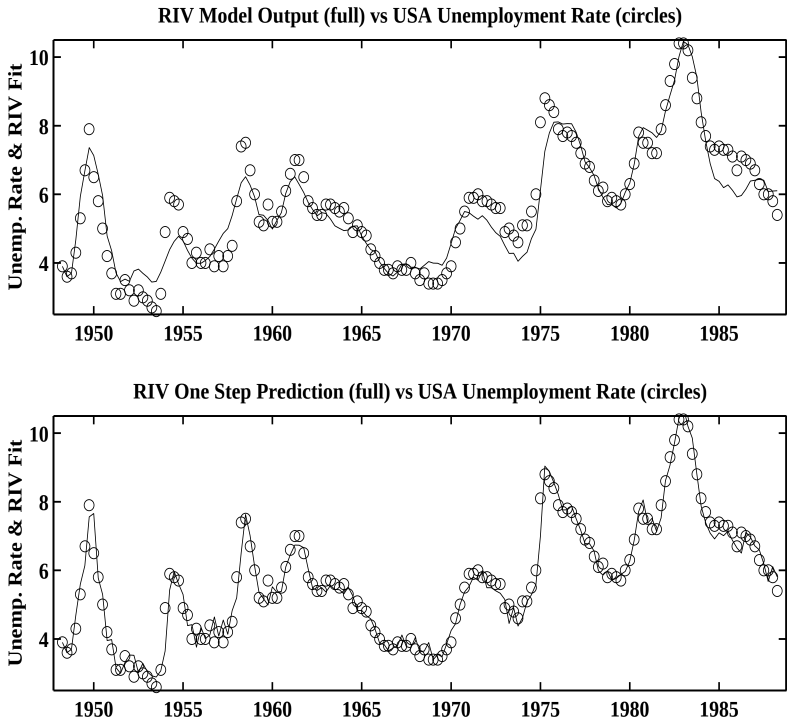

Fig. 7 Final RIV estimation: model output response $\hat{y}_t$ from equation (2) (top) and one step ahead predictions (below), compared with unemployment rate series y_t (circles).

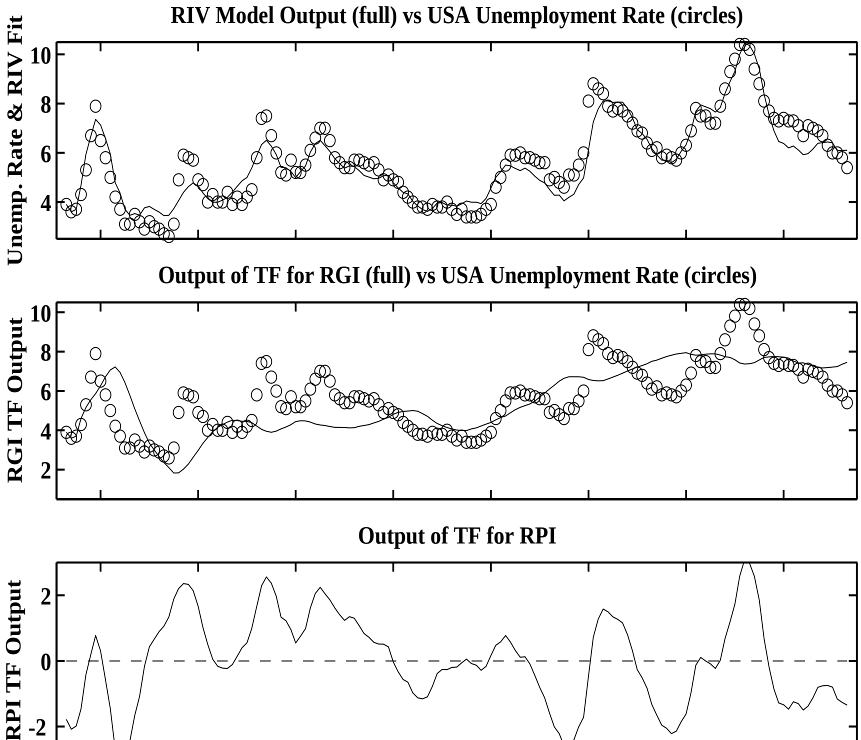

Fig. 8 Final RIV estimation: contributions of the outputs from the TF's in equation (2) associated with each input variable (lower two plots) to the total model output $\hat{y}_t$ (top)

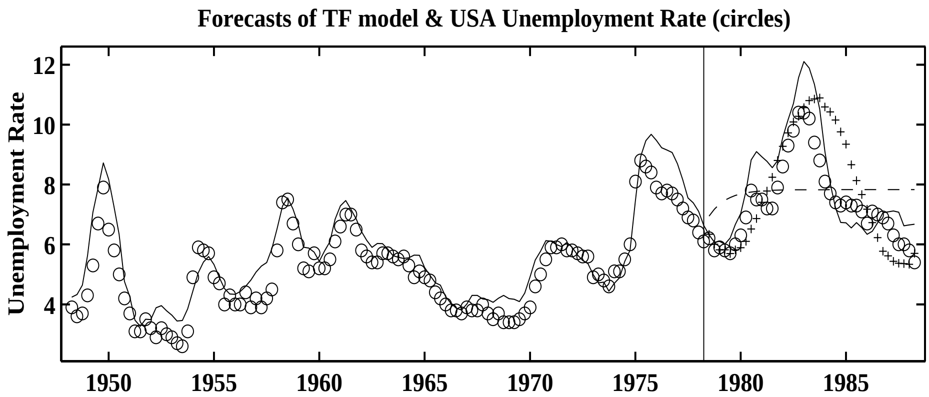

Fig. 9 Unemployment Rate y_t (circles) compared with forecasts of the model (1) when : (a) the actual values of the explanatory variables are used into the future (full); (b) they are forecast using DHR models (crosses); (c) they are assumed to remain constant at their local mean level before the forecast origin (dashed).

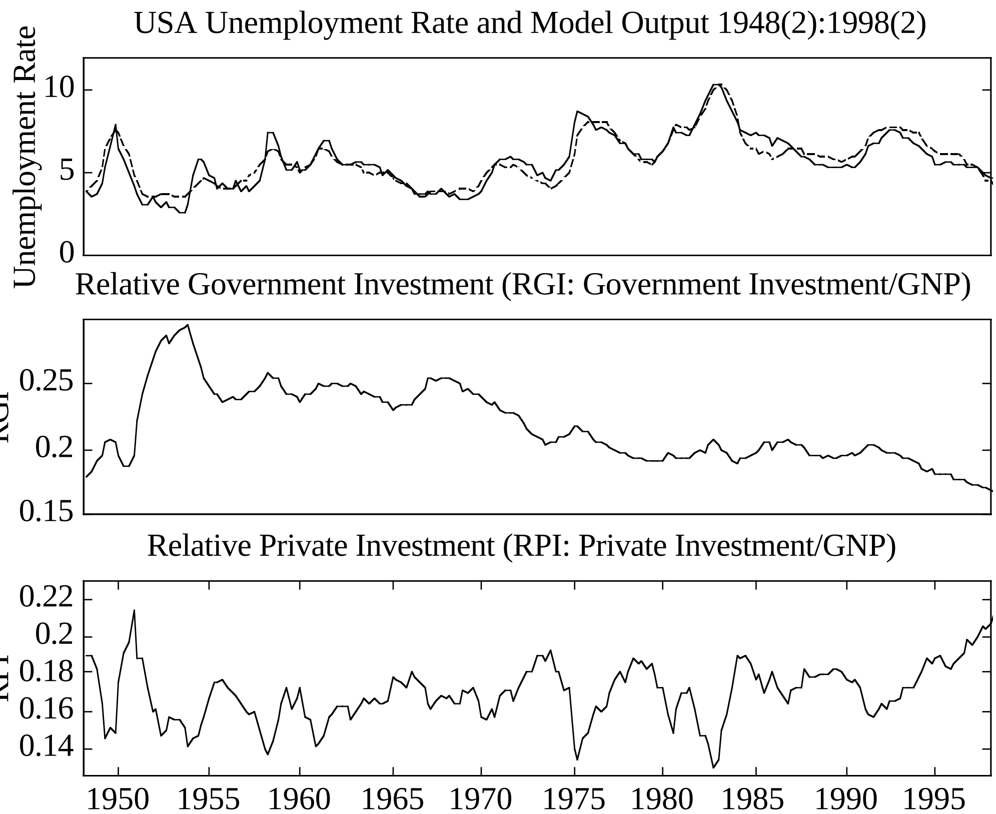

Fig. 10 Unemployment Rate y_t (upper), Relative Government Investment RGI_t (middle) and Relative Private Investment RPI_t (lower) over the period 1948-1998. The dashed line in the upper plot is the model output $\hat{y}_t$ from equation (2).

Parameter	Estimate	SE	T statistic
c	10.178	0.901	11.29
a_1	-0.777	0.028	28.12
b_1	-15.081	1.734	8.68
b_2	-32.981	2.758	11.96
c_1	-1.016	0.077	13.14
c_2	0.246	0.077	3.18

$\sigma^2 =$ 0.1015 R_T^2 =0.894 R^2 =0.965

Jarque-Bera : 1.031 P-Value= 0.59

Q(4) : 1.889 P-Value= 0.17

Q(8) : 8.961 P-Value= 0.01

Steady State Gains: G_1=-67.64; G_2 =-147.92; Time constant, T_c =3.96 quarters

Table 1: Estimated TF model between the ratios to GNP of Private and Public Investment (RPI_t , RGI_t) and the unemployment rate y_t . The noise is modelled as an AR(2) process.

$$A_1 = \begin{bmatrix} 1.1240 & 9.1082 & -19.840 \\ 0.0008 & 1.5361 & 0.1789 \\ 0.0030 & -0.5346 & 0.7130 \end{bmatrix} \qquad dA_1 = \begin{bmatrix} 0.0967 & 9.9504 & 6.3934 \\ 0.0009 & 0.0954 & 0.0613 \\ 0.0018 & 0.1804 & 0.1159 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.4080 & -47.0183 & -17.0804 \\ 0.0009 & -0.4023 & -0.0544 \\ 0.0042 & 0.6690 & 0.2412 \end{bmatrix} \qquad dA_2 = \begin{bmatrix} 0.1348 & 16.9338 & 8.0978 \\ 0.0013 & 0.1624 & 0.0777 \\ 0.0024 & 0.3070 & 0.1468 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.1241 & 29.2657 & 22.3349 \\ -0.0008 & -0.1673 & -0.0542 \\ -0.0006 & -0.1470 & -0.1946 \end{bmatrix} \qquad dA_3 = \begin{bmatrix} 0.0870 & 9.9532 & 6.2856 \\ 0.0008 & 0.0955 & 0.0603 \\ 0.0016 & 0.1804 & 0.1139 \end{bmatrix}$$

$$\boldsymbol{c} = \begin{bmatrix} 6.0704 \\ 0.4526 \\ 0.2944 \end{bmatrix} \qquad d\boldsymbol{c} = \begin{bmatrix} 1.6251 \\ 0.0156 \\ 0.0295 \end{bmatrix} \qquad \Sigma_e = \begin{bmatrix} 0.18072 & 0.00082 & -0.0019 \\ 0.00082 & 0.000033 & -0.000024 \\ -0.0019 & -0.000024 & 0.000055 \end{bmatrix}$$

$R_1^2 = 0.938$ $R_2^2 = 0.957$ $R_3^2 = 0.740$

Normality : 8.019 P-Value= 0.23

P(4) : 13.47 P-Value= 0.14

P(8) : 36.92 P-Value= 0.79

Table 2: Estimation results for a VAR(3) model. The variables are the Unemployment rate y_t ; Relative Government Investment RGI_t ; and Relative Private Investment, RPI_t .

Null Hypothesis	F-statistic	P-value	χ-statistic	P-value
RGI_t and RPI_t does not cause y_t	87.6902	0	526.1414	0
RGI_t does not cause y_t	34.4122	0	103.2365	0
RPI_t does not cause y_t	11.6692	0	35.0076	0
y_t does not cause RGI_t and RPI_t	2.7928	0.0112	16.7570	0.0102
y_t does not cause RGI_t	0.6878	0.5599	2.0633	0.5594
y_t does not cause RPI_t	2.0795	0.1021	6.2384	0.1006

Table 3: Results of Granger causality tests on VAR(3) model shown in table 2.

Null Hypothesis	P-value	Result
RGI_t exogenous	0.4150 (0.5194)	not reject (RGI_t exogenous)
RPI_t exogenous	0.7726 (0.3794)	not reject (RPI_t exogenous)
RGI_t and RPI exogenous	0.8366 (0.6582)	not reject (Both exogenous)
RPI_t exogenous	1.8190 (0.1775)	not reject (RPI_t exogenous)
RGI_t exogenous	4.4710 (0.0344)	not reject (1%) (RGI_t exogenous) reject (5%) (RGI_t endogenous)

Table 4: results of Hausman tests. The first three rows correspond to the estimation of equation (1) with two 'noise free' inputs estimated from the VAR(3) model. Rows 4 and 5 shows the same tests when only one of the noise-free inputs is included (RPI_t and RGI_t , respectively).